Utility Pricing Death Spiral

Mathematics in Industry Study Group 2016

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Problem statement

- Utility companies bill customers based on their energy usage.
- A portion of this income is used to maintain their infrastructure.
- When customers reduce their energy usage, the utility loses a portion of this income, but their maintenance costs remain unchanged.

South African context pt. 1

- 50% of households receive subsidized power.
- Certain customers are charged higher tariffs in order to subsidise the provision of electricity to low-income groups.
- These high-tariff customers are increasingly likely to reduce their energy usage by using alternative energy sources.
- This loss of income forces the utility to increase tariffs and creates a further incentive to move to alternative energy sources.

South African context pt. 2

- The distributed network is fragmented.
- There are 120 municipalities with less then 1000 customers.
- More then 90 municipalities receive less than R1 million in revenue.
- The utility runs the risk of being unable to maintain its infrastructure.

A fair solution?

- The national grid is an important resource regardless of how energy is generated.
- Customers using alternative sources of energy still benefit from access to the national grid.

A fair solution?

Other options to cover the cost of grid infrastructure:

- Tax funds.
- Grid connection fees.

General model

We consider a general model in which the utility's customers are divided into n different tariff groups. We define the following variables:

- G_i is the number of customers in tariff group i.
- G_{n+1} is the number of potential customers who do not have access to the grid.
- A is the number of customers who supplement their energy with alternative sources.
- U_i is the energy usage for the i^{th} group, where $i \in \{1, ..., n, A\}$.
- T_i is the electricity tariff per unit of electricity of group G_i. Here $T_n < T_{n-1} < \cdots < T_1$
- T_A is the electricity tariff for group A.
- C_i is the cost for the i^{th} group per unit of electricity.
- H_i is the fixed grid connection fee for the i^{th} group G_i.

Population dynamics

The number of customers in each tariff group changes due to a number of different factors.
These groups are modelled by the following equations:

$$\frac{dA}{dt} = \sum_{i=1}^{n} (\chi_i(t) + r_i G_i K(t) g(t) + m_i G_i A),$$

$$\frac{dG_i}{dt} = \overline{\beta_i} \cdot \overline{G} - \chi_i(t) - \overline{r_i} \cdot \left(\frac{A}{G} \right) K(t) g(t) - m_i G_i A.$$

The $\overline{\beta_i}$ coefficients are related to economic growth and specify how quickly customers are increasing their electricity usage and moving to higher tariff groups.

Supply shortage

The $\overline{r_i}$ coefficients represent the possibility of a customer moving to a different tariff group (or alternative energy) when the utility is failing to keep up with their current demand for energy.
Here,

$$K(t) = \frac{\sum_{i=1}^{n} G_i U_i + \alpha A U_A}{k(t)} - 1,$$

and

$$g(t) = \begin{cases} 1, & K(t) > 0, \\ 0, & \text{Otherwise.} \end{cases}$$

Tariff increases and interaction term

The $\chi_i(t)$ terms describe the rate at which customers switch to alternative energy due to tariff increases.
Here,

$$\chi(t) = \left(\frac{G_1 - g_0}{q}\right) \sigma \left(1 - \frac{a(t)}{T_1} - \gamma\right),$$

and

$$\sigma(z) = \frac{1}{1 + e^{-z}}.$$

The m_i terms describe the rate at which customers switch to alternative energy as a result of increasing awareness and popularity.

Death spiral conditions

The utility death spiral occurs when the rate of economic growth $\overline{\beta_i}$ is too low to compensate for the effects of higher tariffs, unreliable supply and increased awareness of alternative energy sources.

$$\frac{dG_i}{dt} = \overline{\beta_i} \cdot \overline{G} - \chi_i(t) - \overline{r_i} \cdot \left(\frac{A}{G} \right) K(t) g(t) - m_i G_i A.$$

Profit function

Using this information, we can calculate the utility's revenue and expenses:

$$\text{Profit} = \overbrace{\varepsilon}^{\substack{\text{External}\\ \text{Funding}}} + \overbrace{\sum_{i=1}^{n} G_i \left(U_i T_i + H_i\right) + \alpha A T_A U_A + A H_A}^{\text{Revenue}}$$
$$- \underbrace{\left[F(t) + \sum_{i=1}^{n} \left(G_i U_i C_i\right) + \alpha A C_A U_A\right]}_{\text{Total Cost}}.$$

Strategies

The utility needs to ensure that their profit never drops below 0 in order to maintain their infrastructure.

$$\text{Profit} = \overbrace{\varepsilon}^{\substack{\text{External}\\\text{Funding}}} + \overbrace{\sum_{i=1}^{n} G_i \left(U_i T_i + H_i\right)}^{\text{Core Revenue}} + \overbrace{\alpha A T_A U_A + A H_A}^{R_A}$$
$$- C_{total}.$$

We can ensure that this condition is met by using the profit equation to adjust T_i, H_i and ε.

External funding

One strategy is for the external funding to be defined as follows:

$$\varepsilon = -\left[\sum_{i=1}^{n} G_i \left(U_i T_i + H_i \right) + R_A \right] + C_{total}.$$

If all tariffs are kept constant, any decrease in income must be covered by external funding.

Tariff increase

We define the highest tariff price as follows:

$$T_1 = \frac{-\varepsilon - \sum_{i=2}^{n} G_i \left(U_i T_i \right) - \sum_{i=1}^{n} G_i H_i - R_A + C_{total}}{U_1 G_1}.$$

The tariff for G_1 is increased to cover the drop in income.

Grid fee increase

We can alternatively define the grid fee:

$$H_1 = \frac{-\varepsilon - \sum_{i=1}^{n} G_i \left(U_i T_i\right) - \sum_{i=2}^{n} G_i H_i - R_A + C_{total}}{G_1}.$$

The grid fee for G_1 is increased to cover the drop in income.

Distributed tariff increase

Tariffs are increased by $\mu_i Y$ to cover the drop in income, where μ_i are weights that determine how much each group will be affected by the increase.

$$0 = \varepsilon + \sum_{i=1}^{n} G_i U_i \left(T_i + \mu_i Y\right) + \sum_{i=1}^{n} G_i H_i + \alpha A U_A (T_A + \mu_A Y) + A H_A - C_{total},$$

$$Y = \frac{-\varepsilon - \sum_{i=1}^{n} G_i (U_i T_i + H_i) - \alpha A U_A T_A - A H_A + C_{total}}{\sum_{i=1}^{n} G_i U_i \mu_i + \alpha A U_A \mu_A}.$$

Distributed grid fee increase

Grid fees are increased by $\omega_i Y$ to cover the drop in income, where ω_i are weights that determine how much each group will be affected by the increase.

$$0 = \varepsilon + \sum_{i=1}^{n} G_i U_i T_i + \sum_{i=1}^{n} G_i \left(H_i + \omega_i Y\right) + \alpha A T_A U_A,$$
$$+ A(H_A + \omega_A Y) - C_{total},$$

$$Y = \frac{-\varepsilon - \sum_{i=1}^{n} G_i(U_i T_i + H_i) - \alpha A T_A U_A - A H_A + C_{total}}{\sum_{i=1}^{n} G_i \omega_i + A\omega_A}.$$

Distributed tariff and grid fee increase

Grid fees are increased by $\mu_i Y$ and Tariffs by $\omega_i Y$. In this case, both strategies are implemented.

$$0 = \varepsilon + \sum_{i=1}^{n} G_i(U_i\left(T_i + \mu_i Y\right) + H_i + \omega_i Y)$$
$$+ \alpha A U_A(T_A + \mu_A Y) + A(H_A + \omega_A Y) - C_{total},$$

$$Y = \frac{-\varepsilon - \sum_{i=1}^{n} G_i(U_i T_i + H_i) - \alpha A T_A U_A - A H_A + C_{total}}{\sum_{i=1}^{n} G_i U_i \mu_i + \sum_{i=1}^{n} G_i \omega_i + \alpha A U_A \mu_A + A\omega_A}.$$

Example

Our population dynamics are governed by:

$$
\begin{aligned}
\frac{dA}{dt} &= \chi(t) + r_1(G_1 - G_m)K(t)g(t) + m(G_1 - G_m)A, \\
\frac{dG_1}{dt} &= \beta_2 G_2 - \chi(t) - r_1(G_1 - G_m)K(t) - m(G_1 - G_m)A, \\
\frac{dG_2}{dt} &= -\beta_2 G_2 + \beta_3 G_3 - r_2 G_2 K(t), \\
\frac{dG_3}{dt} &= \beta_4 G_4 - \beta_3 G_3 - r_3 G_3 K(t) + r_2 G_2 K(t) g(t), \\
\frac{dG_4}{dt} &= -\beta_4 G_4 + r_3 G_3 K(t) g(t) + r.
\end{aligned}
$$

Example

Here,

$$K(t) = \frac{\sum_{i=1}^{n} G_i U_i + \alpha A U_A}{k(t)} - 1,$$

$$g(t) = \begin{cases} 1, & K(t) > 0, \\ 0, & \text{Otherwise}, \end{cases}$$

$$\chi(t) = \left(\frac{G_1 - G_m}{q} \right) \sigma \left(1 - \frac{a(t)}{T_1} - \gamma \right),$$

$$\sigma(z) = \frac{1}{1 + e^{-z}}.$$

Governing Equations

We define the tariff for the high-income group, T_1, so that $R = C$. This simplifies to the following equation:

$$T_1 = \frac{F(t) + C_1 G_1 U_1 + \sum_{i=2}^{3} (C_i - T_i) G_i U_i + \alpha C_1 A U_A}{G_1 U_1 + \alpha A U_A}$$

$$- \frac{H_1 \left(G_1 + A\right) + \sum_{i=2}^{3} H_i G_i}{G_1 U_1 + \alpha A U_A}.$$

Example results

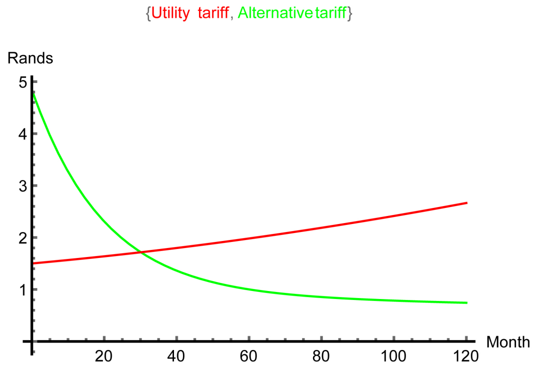

Figure: Comparison of utility tariff against alternative tariff over time. Here, $\gamma = 0.1$, $\alpha = 0.05$, $G_m = 460000$, $r_1 = 0.000086$, $r_2 = 0.000112$, $r_3 = 0.000134$, $\beta_2 = 0.000021$, $\beta_3 = 0.000067$, $\beta_4 = 0.0007$. A death spiral occurs for these parameters.

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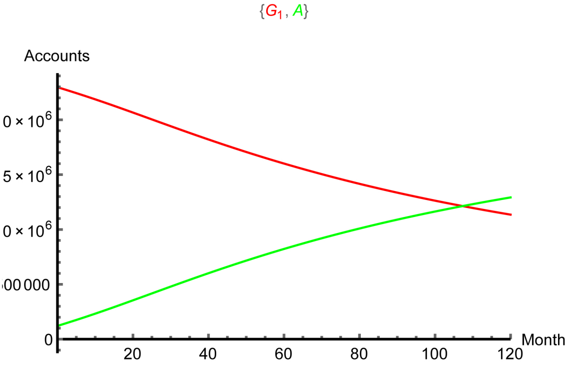

Figure: Comparison of utility high-tariff users against alternative energy users over time. Here, $\gamma = 0.1$, $\alpha = 0.05$, $G_m = 460000$, $r_1 = 0.000086$, $r_2 = 0.000112$, $r_3 = 0.000134$, $\beta_2 = 0.000021$, $\beta_3 = 0.000067$, $\beta_4 = 0.0007$. A death spiral occurs for these parameters.

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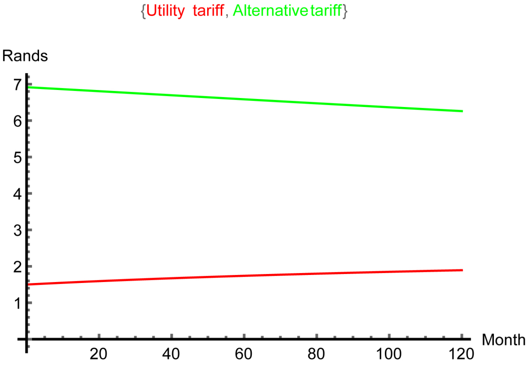

Figure: Comparison of utility tariff against alternative tariff over time. Here, $\gamma = 0.1$, $\alpha = 0.5$, $G_m = 1500000$, $r_1 = 0.000086$, $r_2 = 0.000112$, $r_3 = 0.000134$, $\beta_2 = 0.000021$, $\beta_3 = 0.000067$, $\beta_4 = 0.0007$. A death spiral is avoided for these parameters.

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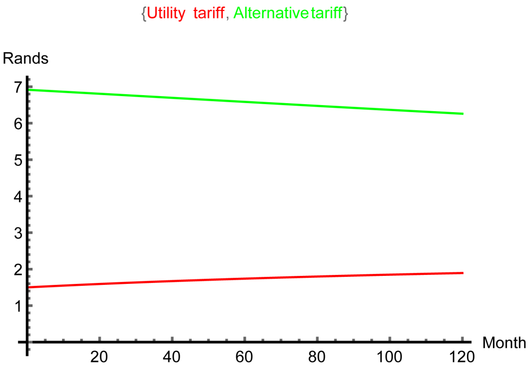

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Conclusion

- Economic growth is crucial.
- Addressing short-term concerns such as load-shedding and grid access can help to mitigate the effect of cheaper alternative energy.
- The usage-based tariff structure is problematic and unsustainable in the long term.